We present what we believe to be a new digital holographic imaging method that is able to determine simultaneously the distributions of intensity, phase, and polarization state at the surface of a specimen on the basis of a single image acquisition. Two reference waves with orthogonal polarization states interfere with the object wave to create a hologram that is recorded on a CCD camera. Two wave fronts, one for each perpendicular polarization state, are numerically reconstructed in intensity and phase. Combining the intensity and the phase distributions of these two wave fronts permits the determination of all the components of the Jones vector of the object-wave front. We show that this method can be used to image and measure the distribution of the polarization state at the surface of a specimen, and the obtained results indicate that precise quantitative measurements of the polarization state can be achieved. An application of the method to image the birefringence of a stressed polymethyl methacrylate sample is presented. © 2002 Optical Society of America

1. Introduction

Polarization imaging is useful for revealing inner structures or stresses of materials. Different methods have been developed to determine the polarization state. For example, we can mention some recent studies in optical coherence tomography, real-time polarization phase-shifting system, polarization contrast with near-field scanning optical microscopy, and Pol-Scope, among others. These techniques require several measurements for determining the polarization state. Typically several acquisitions for different orientations of birefringent optical components such as polarizers, half-wave ($\lambda/2$), and quarter-wave ($\lambda/4$) plates are needed to obtain the Jones parameters. We propose here a new approach, based on digital holographic imaging (DHI) that requires a single acquisition.

In a recent paper, it was shown that when the configuration initially proposed by Lohmann is used, digital holography can be used as a polarization imaging technique. The basic idea is to create a hologram of the specimen by producing the interference between the object wave and two reference waves that have perpendicular polarization states. The reconstruction of such a hologram produces two wave fronts, one for each reference wave, or, in other words, one for each perpendicular polarization state. The implementation of this idea in digital holography consists of using a numerical method to reconstruct the hologram, which is digitally recorded by a CCD camera. The method presented in Ref. 8 takes advantage of only the reconstructed amplitude distributions and therefore provides only two of the four Stokes vector components. Here we apply the reconstruction procedure described in Ref. 7 to reconstruct the phase information as well as the amplitude. We demonstrate that comparing the amplitude and the phase distributions associated with the two orthogonal polarization states enables us to determine entirely the distribution of polarization states at the surface of the specimen. An attractive feature of the method presented here is that the polarization information is obtained simultaneously with the amplitude and the phase information. This provides a complete description of the optical field at the surface of the specimen on the basis of a single image acquisition performed at video frequency.
2. Method and Experiments

The configuration used for hologram recording is shown in Fig. 1(a). It consists of a modified Mach–Zehnder interferometer with two reference waves, \( R_1 \) and \( R_2 \). A He–Ne laser (20 mW) is used to generate the three interfering waves, \( R_1, R_2, \) and \( O \). In each arm, a polarizer (Pol.) and a half-wave plate (\( \lambda/2 \)) are used to control the polarization and the intensity of each wave. The polarizers (Glan Laser Polarizers made of calcite with an extinction ratio of 200,000:1) are oriented to obtain horizontal and vertical linear polarization for \( R_1 \) and \( R_2 \), respectively. We can choose the polarization of the beam illuminating the object by changing the orientation (\( \delta \)) of the polarizer and of the quarter-wave plate (\( \lambda/4 \)) in the object arm.

To achieve precise phase measurements, we have paid particular attention to select optical elements that minimize wave-front distortions. The wave plates (\( \lambda/2 \) and \( \lambda/4 \)), made of crystal quartz, have a wave-front distortion of \( \lambda/10 \) and the polarizer a wave-front distortion of less than \( \lambda/4 \).

The hologram is recorded in off-axis geometry with the three waves propagating along distinct directions. As shown in Fig. 1(b), object wave \( O \) propagates along \( 0z \) with normal incidence on the CCD plane \( 0xy \). Reference waves \( R_1 \) and \( R_2 \) propagate symmetrically with respect to the \( 0xz \) plane (\( \varphi_1 = \varphi_2' \)) and have similar incidence angles (\( \theta_1 \approx \theta_2 \)) with respect to \( 0z \). The interference among \( O, R_1, \) and \( R_2 \) gives rise to the hologram. As \( R_1 \) and \( R_2 \) have orthogonal polarizations, they do not interfere \( (R_1 R_2^* = R_1^* R_2 = 0) \), and the hologram intensity is

\[
I_H(x, y) = |R_1|^2 + |R_2|^2 + |O|^2 + R_1 O^* + R_2 O^* + R_1^* O + R_2^* O.
\]  

The three first terms of Eq. (1) form the zero order of diffraction, the fourth and fifth terms produce two real images corresponding respectively to the horizontal and the vertical polarization states, and the last two terms produce the virtual images. An example of a hologram recorded with this configuration, with a U.S. Air Force resolution test target used as sample, is given in Fig. 2. Two fringe patterns with fixed orientations (\(-38.2^\circ\) and \(44.4^\circ\) with respect to the horizontal) can be observed. The carrier frequency of these two fringe patterns is fixed by \( \theta_1 \) and \( \theta_2 \).

Within the framework of the Jones formalism, the object wave can be expressed as

\[
O = \begin{bmatrix}
O_1(x, y) \exp[i\varphi(x, y)] \\
O_2(x, y) \exp[i\varphi_2(x, y)]
\end{bmatrix} \exp[i\phi_0(x, y)].
\]  

Together, \( O_1(x, y), O_2(x, y), \varphi(x, y), \) and \( \varphi_2(x, y) \) are four functions defining entirely the polarization state of the object wave. \( O_1(x, y) \) and \( O_2(x, y) \) are the amplitudes of the parallel and the perpendicular components, respectively, of the object-wave polarization state, and \( \varphi(x, y) \) and \( \varphi_2(x, y) \) represent the phases of these components. A scalar object wave of intensity \( I_0 = O_1^2 + O_2^2 \) and phase \( \phi_0(x, y) \) can be defined from the definition of Eq. (2). Assuming that \( R_1 \) and \( R_2 \) have the same intensity \( I_R = R_2^* \) and that the orientation of their linear polarization states form an orthogonal base, we can write

\[
R_1 = \begin{bmatrix} R \exp(i\varphi_1) \\ 0 \end{bmatrix} \exp(i k_1 \cdot r),
\]

\[
R_2 = \begin{bmatrix} 0 \\ R \exp(i\varphi_2) \end{bmatrix} \exp(i k_2 \cdot r).
\]
where \( \varphi_1 \) and \( \varphi_2 \) are two constants describing the fact that the optical path length is not the same for the two reference waves. If we consider the two virtual images [sixth and seventh terms of Eq. (1)], introducing Eqs. (2) and (3) into Eq. (1) gives

\[
R_1^*O = RO_1(x, y) \exp[i(\varphi_1(x, y) - \varphi_1)] \\
\times \exp[i(\phi_0(x, y) - k_1 \cdot r)]
\]

for the horizontal polarization state and

\[
R_2^*O = RO_\perp(x, y) \exp[i(\varphi_\perp(x, y) - \varphi_2)] \\
\times \exp[i(\phi_0(x, y) - k_2 \cdot r)]
\]

for the vertical polarization state.

In digital holography, the hologram intensity is transmitted to a computer by a framegrabber board in the form of a digital hologram \( I_H(k, l) \) that results from the two-dimensional spatial sampling of \( I_H(x, y) \) by the CCD camera:

\[
I_H(k, l) = I_H(x, y) \operatorname{rect} \left[ \frac{x}{L}, \frac{y}{L} \right] \\
\times \sum_{k=-N/2}^{N/2} \sum_{l=-N/2}^{N/2} \delta(x - k \Delta x, y - \Delta y),
\]

where \( L \) is the dimension of the detector, \( \Delta x \) and \( \Delta y \) are the sampling intervals in the hologram plane (detector pixel size), \( N \) is the number of pixels along each direction of the CCD camera, \( k \) and \( l \) are integers \((-N/2 \leq k, l \leq N/2)\), \( \delta \) is the Dirac delta function, and \( \operatorname{rect}[x/L, y/L] \) is a function equal to 1 inside the detector area and equal to zero elsewhere. The sensitive part (area \( L \times L = 4.85 \times 4.85 \text{ mm} \)) of the CCD camera (Hitachi Denshi KPKM2) records a hologram of \( N \times N = 512 \times 512 \) pixels.

The numerical reconstruction of the digital hologram is performed within the framework of the Fresnel approximation with the following algorithm:

\[
\Psi(m, n) = A \exp \left[ \frac{i\pi}{\lambda d} (m^2 \Delta x^2 + n^2 \Delta y^2) \right] \\
\times \text{FFT} \left\{ R_m(k, l) I_H(k, l) \right\} \\
\times \exp \left[ \frac{i\pi}{\lambda d} (k^2 \Delta x^2 + l^2 \Delta y^2) \right]_{m,n},
\]

where \( m \) and \( n \) are integers \((-N/2 \leq m, n \leq N/2)\), \( \Psi(m, n) \) is an array of complex numbers describing the reconstructed wave front in the observation plane \( 0z_\eta \). FFT is the fast Fourier transform operator, \( \lambda \) is the wavelength, and \( A = \exp(i2\pi d/\lambda)/(i\pi \lambda) \) is a constant. The \( d \) parameter, appearing in the two quadratic phase terms before and after the FFT, represents the distance at which the Fresnel diffraction pattern is evaluated (the distance between \( 0z_\eta \) and \( 0z_\eta \)). The adjustment of the \( d \) value is related to image focusing \( R_m \) is a computed array of complex numbers called the digital reference wave and is defined below. \( \Delta x \) and \( \Delta y \) are the sampling intervals in the observation plane, which are calculated as follows:

\[
\Delta x = \Delta y = \frac{\lambda d}{NAx} = \frac{\lambda d}{L}.
\]

If we compute the modulus of the reconstructed wave front \( I(m, n) = |\text{Re}[\Psi(m, n)]|^2 + |\text{Im}[\Psi(m, n)]|^2 \)\(^{1/2} \) we obtain an amplitude-contrast image that represents the distribution of the optical amplitude in the observation plane. Figure 3(a) presents the amplitude-contrast image obtained by numerical reconstruction of the hologram presented in Fig. 2. Five distinct terms appear at different locations in the observation plane. The zero order of diffraction \( Z = [R_1^*O]^2 + [R_2^*O]^2 + [O]^2 \) produces a square and uniform contribution in the center of the image. Symmetrically with respect to the center, two pairs of twin images (one for each polarization state) can be observed. \( R_1^*O \) and \( R_2^*O \) produce two virtual images, and \( R_1^*O \) and \( R_2^*O \) produce two real images. The disposition of these different terms in the observation plane is a consequence of the off-axis geometry presented in Fig. 1(b). The virtual images are in focus in the image presented in Fig. 3(a) because it has been obtained for a negative reconstruction distance \( d = -22.1 \text{ cm} \). The real images can be reconstructed in focus by use of a positive reconstruction distance. Figure 3(b) presents the reconstructed amplitude-contrast image obtained after application of the digital image processing method described in Ref. 10. The hologram intensity is Fourier transformed, and a filtering process is applied in the Fourier plane to suppress spatial frequencies corresponding to the.
zero order and to the real images. As explained in Ref. 10, the noise produced by some parasitic interferences can also be eliminated by this procedure. Another digital image processing method is applied to the hologram before its numerical reconstruction to suppress fluctuations of the reconstructed field that are due to the finite size of the CCD aperture. This method is described in Ref. 11 and consists of a digital apodization of the hologram aperture.

A phase-contrast image that represents the two-dimensional distribution of the optical phase at the surface of the specimen can be obtained by computation of the argument of the reconstructed wave front:

\[ \phi(m, n) = \arctan(\text{Im}[\Psi(m, n)]/\text{Re}[\Psi(m, n)]) \]

In this case, as expressed by Eq. (7), a computed array of complex numbers, called the digital reference wave \( R_D(k, l) \), is introduced as a term that multiplies the hologram intensity before the Fresnel transform calculation.\(^7\) The role of this digital reference wave is to simulate the illumination of the hologram by a duplication of the reference wave as performed classically for hologram reconstruction. If we assume a plane wave as reference, \( R_D \) is calculated as follows:

\[ R_D(k, l) = \exp \left[ \frac{2i\pi}{\lambda} (k_x k \Delta x + k_y l \Delta y) \right], \quad (9) \]

where \( k_x \) and \( k_y \) are two parameters that must be precisely adjusted so that \( R_D \) fits the reference wave used to create the hologram. As defined by Eq. (9), the digital reference wave is a purely scalar object, which does not take into account the polarization state of the reference wave. It is introduced to compensate for a constant gradient that appears in the reconstructed phase distribution as a consequence of the off-axis geometry.

The procedure for adjustment of \( k_x \) and \( k_y \) is described in Ref. 7. For the present application, we have to define two different digital reference waves: \( R_{D_1} \) for the horizontal polarization state and \( R_{D_2} \) for the vertical polarization state. These two digital reference waves differ only by the values of \( k_x \) and \( k_y \), which are adjusted to have

\[ R_{D_1} = \exp(i\mathbf{k}_1 \cdot \mathbf{r}) = R_1, \quad R_{D_2} = \exp(i\mathbf{k}_2 \cdot \mathbf{r}) = R_2. \quad (10) \]

Let us consider now the product \( \Psi' = R_D I_H \), which represents the reconstructed wave front, defined in the hologram plane. When Eqs. (4), (5), and (10) are taken into account, the two virtual images give two wave fronts defined by

\[ \Psi_{1'} = R_{D_1} R_{I_1}^{*} O \]

\[ = RO_{1}(x, y) \exp[i(\Psi_{1}(x, y) - \phi_1)] \exp[i(\phi_0(x, y))] \]

\[ (11) \]

for the horizontal polarization state and

\[ \Psi_{1'} = R_{D_2} R_{I_2}^{*} O \]

\[ = RO_{2}(x, y) \exp[i(\Psi_{2}(x, y) - \phi_2)] \exp[i(\phi_0(x, y))] \]

\[ (12) \]

for the vertical polarization state. These two wave fronts are defined in the hologram plane, and the Fresnel diffraction calculation [Eq. (7)] describes their propagation to the observation plane \( 0 \xi_0 \eta \). The two wave fronts \( \psi_{1}(\xi, \eta) \) and \( \psi_{2}(\xi, \eta) \) represent, according to the principle of holography, duplicates of the corresponding optical fields at the surface of the specimen. The amplitudes of these two wave fronts are proportional to the amplitudes of the parallel and
ponent in Fig. 4, which gives numerical reconstructions of reconstructed phase distributions. For example, and their arguments give

\[ \text{arg}[\psi_}(\xi, \eta)] = \varphi_1(\xi, \eta) - \varphi_1 + \varphi_2(\xi, \eta), \]  

\[ \text{arg}[\psi_\perp(\xi, \eta)] = \varphi_\perp(\xi, \eta) - \varphi_2 + \varphi_0(\xi, \eta). \]  

With the DHI method presented here, \( RO_\parallel(\xi, \eta) \) and \( RO_\perp(\xi, \eta) \) are obtained from the amplitude-contrast image (see Fig. 3) and the two arguments from the reconstructed phase distributions. For example, in Fig. 4, which gives numerical reconstructions of the hologram shown in Fig. 2, \( \text{arg}[\psi_\parallel(\xi, \eta)] \) can be obtained from Fig. 4(a), which shows the phase dis-

tribution obtained for \( R_D = R_{D1} \) and \( \text{arg}[\psi_\perp(\xi, \eta)] \) from Fig. 4(b), which has been obtained for \( R_D = R_{D2} \).

An important characteristic describing the polarization state is the phase difference between the perpendicular and the parallel components. To define this difference, we introduce two constants, \( \tau_\parallel \) and \( \tau_\perp \), defining the coordinates of a vector \( \tau \) in the observation plane that describes the translation between the locations of \( \psi_\parallel(\xi, \eta) \) and \( \psi_\perp(\xi, \eta) \) (see Fig. 3). We can then define the phase difference between the perpendicular and the parallel components from Eqs. (14):

\[ \text{arg}[\psi_\perp(\xi - \tau_\parallel, \eta - \tau_\parallel)] - \text{arg}[\psi_\parallel(\xi, \eta)] = [\varphi_\perp(\xi - \tau_\parallel, \eta - \tau_\parallel) - \varphi_\parallel(\xi, \eta)] - (\varphi_2 - \varphi_1) = \Delta \varphi_\parallel(\xi^*, \eta^*) - \Delta \varphi_R, \]  

where \( \Delta \varphi_\parallel(\xi^*, \eta^*) \) is the phase difference between the two orthogonal components of the polarization of the object wave. The \( \xi^* \) and \( \eta^* \) coordinates are introduced to describe the fact that the translation of the perpendicular component by \((\tau_\parallel, \tau_\parallel)\) ensures that \( \varphi_\parallel \) and \( \varphi_\perp \) are considered for the same point (of coordinates \((\xi^*, \eta^*)\)) at the surface of the specimen. \( \Delta \varphi_R \) is the phase difference between the two reference waves. As described in next section, \( \Delta \varphi_R \) can be estimated with an appropriate experimental configuration.

Another important characteristic of the polarization state is the quotient \( O_\parallel/O_\perp \). To ensure that the Jones vector amplitudes are considered for the same point at the surface of the object, this quotient is determined as follows:

\[ \left| \psi_\parallel(\xi - \tau_\parallel, \eta - \tau_\parallel)/\psi_\parallel(\xi, \eta) \right| = O_\parallel(\xi^*, \eta^*)/O_\parallel(\xi^*, \eta^*). \]  

It is important to point out the fact that, as the reconstructed wave front is the result of a numerical calculation, all measured quantities are available in digital form: \( RO_\parallel(m, n), RO_\perp(m, n), \) and \( \Delta \varphi(m, n) \) defined with spatial sampling intervals given by Eq. (8). Here, the translation vector \( \tau \) is defined with the same sampling interval and the superposition of the two virtual images is made with a precision of one pixel. This may introduce errors in the obtained results that could be avoided if, for example, an image translation method based on a spline interpolation is used.

In summary, the digital holographic method presented here allows for the measurements of two characteristics of the Jones vector of the object wave, the phase difference \( \Delta \varphi_\parallel = \varphi_\parallel - \varphi_\perp \), and the ratio \( O_\parallel/O_\perp \). Even if the absolute value of \( O_\perp \) and \( O_\parallel \) cannot be determined because they are multiplied by the reference-wave amplitude \( R \), the polarization information can be obtained. For example, \( \Delta \varphi_\parallel = 0 \) or \( \pi \) corresponds to a linear polarization, \( \Delta \varphi_\parallel = \pi/2 \) and \( O_\parallel/O_\perp = 1 \) to a circular right polarization, \( \Delta \varphi_\parallel = -\pi/2 \) and \( O_\parallel/O_\perp = 1 \) to a circular left polarization,
Fig. 5. Polarization ellipse. The ellipticity is defined by the ratio of the length of the semimajor axis to the length of the semimajor axis, \( b/a = \tan(\omega) \). The ellipse is further characterized by its azimuth \( \alpha \), measured counterclockwise from the \( x' \) axis. \( O_\parallel \) and \( O_\perp \) are the maximum amplitudes of the \( x' \) and the \( y' \) components, respectively, of the electric vector, and \( \tan(\beta) \) is defined by their ratio [see Eqs. (17)].

Intermediate values correspond to elliptical polarization states for which the essential characteristics can be measured. For example, when the polarization ellipse presented in Fig. 5 is considered, the parameters \( \alpha \) (polarization ellipse azimuth), \( \beta \), and \( \tan(\omega) \) (ellipticity), given by the following equations,

\[
\begin{align*}
\alpha &= 2^{-1} \arctan[2 O_\parallel O_\perp \cos(\Delta \varphi)/O_\parallel^2 - O_\perp^2], \\
\beta &= \arctan(O_\perp/O_\parallel), \\
\omega &= 2^{-1} \arcsin[\sin(2\beta)\sin(\Delta \varphi)], \\
\end{align*}
\]
(17)
can be determined because \( R \) is simplified when we replace the theoretical values \( O_\parallel \) and \( O_\perp \) with the measured values \( RO_\parallel \) and \( RO_\perp \).

3. Results

To evaluate the measurement of the amplitudes \( O_\parallel \) and \( O_\perp \), an object composed of two polarizers, presented in Fig. 6, is used. The first polarizer, P1, has a variable orientation \( \delta \), and the second polarizer, P2, has a fixed orientation (\( -45^\circ \)). This object is illuminated by a circularly polarized wave produced by a quarter-wave plate, \( \lambda/4 \), oriented at \( 45^\circ \) with respect to the linearly polarized wave from the laser source. The entire illumination wave passes through P1 but only a part (part B in Figs. 6 and 7) passes through P2. Reconstructed amplitude-contrast images obtained for three different values of \( \delta \) (0°, 45° and 90°) are shown in Figs. 7(a)–7(f). Only the virtual images are presented. Images in the left-hand column correspond to the parallel polarization state and images in the right-hand column to the perpendicular polarization state. Figures 7(a) and 7(b) present the results obtained for \( \delta = 0^\circ \), Figs. 7(c) and 7(d) for \( \delta = 45^\circ \), and Figs. 7(e) and 7(f) for \( \delta = 90^\circ \). In part B of the images, the amplitudes are identical for both orthogonal states for any \( \delta \) value because P2 is oriented at \( -45^\circ \). In part A, the magnitude of the parallel

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![Figure 5](image5.png)

![Figure 6](image6.png)

![Figure 7](image7.png)
amplitudes can be observed for both polarization states when \( P_1 \) is oriented at 45°. In this case, the intensities in part B are near zero because the relative orientation of \( P_1 \) and \( P_2 \) is 90°.

The results of Fig. 7 demonstrate qualitatively that the method behaves as expected for a well-known particular situation. If we now consider intermediate \( \delta \) values, the measured amplitude in part A is theoretically given by the product between the Jones matrix of a linear polarizer and the Jones vector of a right circularly polarized wave:

\[
O = \frac{1}{\sqrt{2}} \begin{bmatrix} \cos^2 \delta & \sin \delta \cos \delta \\ \sin \delta \cos \delta & \sin^2 \delta \end{bmatrix} \begin{bmatrix} 1 \\ i \end{bmatrix}
\]

\[
= \frac{1}{\sqrt{2}} \begin{bmatrix} \cos \delta \\ \sin \delta \exp(i\delta) \end{bmatrix}.
\]

As \( O_1 \) and \( O_\perp \) represent amplitudes that are positive by definition, Eq. (18) can be rewritten as

\[
O = \frac{1}{\sqrt{2}} \begin{bmatrix} |\cos \delta| \\ |\sin \delta| \exp(i\gamma) \end{bmatrix} \exp(i\delta)
\]

\[
= \frac{1}{\sqrt{2}} \begin{bmatrix} O_1 \\ O_\perp \exp(i\gamma) \end{bmatrix} \exp(i\delta),
\]

where \( \gamma \) is 0° if \( \delta \) is in the interval \([0; \pi/2] \cup [\pi; 3\pi/2]\) and \( \pi \) elsewhere. It must also be noted that Eq. (19) gives normalized amplitudes. Figure 8 presents a comparison between theoretical [Eq. (19)] and experimental values for different orientations \( \delta \) of the polarizer.

To evaluate now the measurement of the phase difference \( \Delta \phi_{ot} \), we take a quarter-wave plate oriented at an angle \( \delta = 0° - 180° \), illuminated by a linearly polarized wave oriented at 45°, as the object (Fig. 9). To estimate the value of \( \Delta \phi_{ot} \) [Eq. (15)], we create a reference area in the image by placing a linear polarizer oriented at 45° in a part of the beam after the quarter-wave plate. In this area (part B in Figs. 9 and 10), the theoretical phase difference between the components of the object wave is zero. The reconstructed phase-difference distribution presented in Fig. 10 permits us therefore to determine the phase difference \( \Delta \phi_R \) between the two reference waves by measuring the phase difference in part B. In the other part of the object wave (part A in Figs. 9 and 10) the measured phase difference is given by Eq. (15), and \( \Delta \phi_{ot} \) can be obtained when the measured value of \( \Delta \phi_R \) is added. Theoretically the Jones vector for the light emerging from a quarter-wave plate oriented at 45° and illuminated by a linearly polarized wave at 45° is written as

\[
\frac{\sqrt{2}i \sin \delta \cos \delta}{\cos \delta + \sin \delta \sin \delta} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.
\]

The function describing \( \Delta \phi_{ot} \), derived from Eq. (20) in relation to \( \delta \), is reported in the plot of Fig. 11, which also presents experimental data for comparison.

To illustrate its polarization imaging capabilities, the method has been applied to study a polymethyl methacrylate (PMMA) plane for which a birefringence can be induced by application of a constraint. Figure 12 presents the setup used to create the stressed PMMA sample. The illuminating beam (circle) is polarized linearly at 45°, and a part of it (part A) passes through the sample while another (part B), serving as reference area, passes outside the sample. Examples of results obtained with this sample are given in Fig. 13. Part A, delimited by white lines, is the PMMA sample, and part B is the reference area. The arrows indicate the compression points. Figures 13(a) and 13(b) represent the spatial distributions of \( R_{ot} \) and \( R_{ot} \), respectively. The spatial polarization state is obtained by computation of the spatial distribution of \( \beta(\xi', \eta') \), which is defined in Eqs. (17) and presented in Figure 13(c). Figures 13(d) and 13(e) present the reconstructed phase distributions corresponding to the parallel and the perpendicular components, respectively, of the polarization state. As the phase distribution is defined as modulo 2π, phase jumps (between −π and π) appear in the images. The birefringence properties of the sample are expressed by a different phase delay for the two components of the object wave and are imaged by the spatial distribution of \( \Delta \phi_{ot} \), presented in Fig. 13(f). We obtain \( \Delta \phi_{ot} \) by subtracting Fig. 13(d) from Fig. 13(e) and by adding the phase difference between the reference waves \( \Delta \phi_R \) measured in the reference area.

4. Discussion

Obtained results show that DHI with two reference waves of perpendicular polarization states can be used to obtain the distributions of the Jones vector at the surface of the specimen with a single image acquisition. Even if qualitative results illustrating the principle and the imaging capabilities of the method are rapidly obtained, its quantitative validation is more difficult because several sources of noise and artifacts influence the measurements. The method proposed here combines the standard experimental difficulties of polarization measurements with those of interferometry and is therefore sensitive to external perturbations such as mechanical vibrations, thermal drifts, and air movements. The precision of the measure-
ments depends on the quality of the interfering wavefronts and on the precision of their polarization states' adjustment. For this reason, high-quality optics components introducing low wave-front distortion and with well-defined birefringent behavior must be used. An important source of noise comes from parasitic reflections occurring at optical interfaces in the setup. Even image processing methods (see Ref. 10) and antireflection coatings can reduce these effects; it remains difficult to suppress them completely because of the inherently high sensitivity of interferometric methods to low light intensities. Because of the high coherence of the light source, this parasitic light interferes with the reference and the object waves, and its main effect is that the measured values of $O_{\parallel}$ and $O_{\perp}$ are never equal to zero, as can be seen in Fig. 8.

Fig. 8. Theoretical (solid curve) and experimental (circles) values for $O_{\parallel}$ and $O_{\perp}$ as functions of the orientation $\delta$ of polarizer P1.

Fig. 9. Setup in the object arm for the results presented in Figs. 10 and 11. $\lambda/4$ is a quarter-wave plate oriented at an angle $\delta$; P1 and P2 are polarizers oriented at 45°. Part B, which crosses P2, serves as reference area for measuring the phase difference between the two reference waves.

Fig. 10. Image of phase difference induced by a quarter-wave plate. A is an area in the object surface (quarter-wave plate), and B is an area in the reference part used to estimate $\Delta \varphi_B$. We calculate the effective value of $\Delta \varphi_{\parallel}$ by subtracting $\Delta \varphi_B$ from the phase difference measured in A.
Another crucial point is that the two polarizers in the reference arms must be very carefully adjusted so that the linear polarization states of $R_1$ and $R_2$ form a perpendicular base. In particular, this adjustment must take into account the fact that the reflection by a beam splitter slightly modifies the polarization state. In Fig. 8, we can see that the correlation between experimental and theoretical data is better for $O$ than for $O_s$. This is because $R_1$ undergoes one reflection fewer than $R_2$ before reaching the CCD camera.

The adjustment of the three half-wave plates, in particular that in the object arm (Fig. 1), is also important and must be ideally performed so that, without object, equal amplitudes and phases are obtained for the two polarization components. As the method is based on an interferometric technique, the adjustment of these three half-wave plates is very important because it fixes the phase relation between the interfering wave fronts. For example, in an extreme case, if a phase difference of $\pi/2$ exists between $O$ and $R_1$, they cannot interfere even if the polarization state of the object wave is not strictly perpendicularly polarized.

For the results presented here, the polarizers and the half-wave plates have been adjusted to optimize phase-difference measurements for which an excellent agreement between theory and experiment has been obtained (see Fig. 11). However, this adjustment is not optimal for amplitude measurements, and, as can be seen in Fig. 8, the maximum of $O$ and the minimum of $O_s$ are shifted by approximately $4^\circ$.

It is important to point out also that the precision of the phase-difference determination depends on the polarization state of the object wave. In an extreme case, if the object wave is horizontally polarized, no interference occurs with $R_2$ and the phase difference is undetermined. Therefore, to obtain maximum
precision, the illuminating beam polarization must be adapted, depending on the sample.

The main drawback of the method is that the presence of a reference area is required for measuring \( \Delta \varphi_R \) (the phase difference between the two reference waves). It cannot be calibrated in the beginning of an experiment because mechanical vibrations, air turbulence, and thermal effects modify this value during the measurement duration. However, the reference area can be avoided for imaging purposes if only relative information is needed.

Results obtained with a stressed PMMA sample (Fig. 13) demonstrate the capacity of the method to provide image of the polarization state at the surface of a specimen. As expected, the stress field induces black curves on Figs. 13(a) and 13(b). These figures could be obtained with a standard polariscope,\(^{12}\) with a first polarizer oriented at 45° (as illuminating beam) and with a second polarizer oriented at 0° and 90° as analyzer. The interesting point in Fig. 13 is that we can observe that the images corresponding to the two orthogonal components are clearly different because of the birefringent property of the sample. Figures 13(c) and 13(f), which compare the amplitude-contrast and the phase-contrast values, show the difference between the two orthogonal components and reveal therefore the birefringence induced by the constraint. Standard polariscopes could obtain these images, but only with several image acquisitions, against only one for the method presented here.

Another characteristic of the method that remains to be discussed is the transverse resolution. As presented here, the implementation without magnifying optics (microscope objective) allows the transverse resolution defined by Eq. (8), which gives the pixel size in the observation plane. Typically resolutions approaching 40 \( \mu \)m can be achieved in this case. However, as presented in Ref. 7, microscopic observations with the same transverse resolution as classical optical microscopy can be achieved with an implementation that includes a microscope objective producing a magnified image of the specimen.

5. Conclusion
The results presented here demonstrate that the distribution of the complete polarization state of the optical field at the surface of a sample can be obtained by numerical reconstruction of a hologram recorded with two reference waves that have orthogonal polarization states. In comparison with other standard techniques for polarization measurements, an attractive feature of this method is that the information can be obtained from one single image acquisi-

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Fig. 13. Images obtained for a strained PMMA sample: (a) amplitude contrast \( R_{Oy} \), (b) amplitude contrast \( R_{Ox} \), (c) amplitude parameter \( \beta \), (d) phase contrast for the horizontal polarization, (e) phase contrast for the vertical polarization, (f) phase difference \( \Delta \varphi \). The arrows indicate the compression points; the white lines delimit the PMMA samples (rectangles).
tion that can be recorded at video frequency. This opens interesting perspectives for the real-time observation of processes involving dynamic changes of polarization states. Another advantage is that the parameters of the Jones vector, which are more intuitive than the Stokes parameters, are directly measured. Using test objects for which theoretical data can be determined, we have demonstrated that precise quantitative measurements can be performed. The method has also been successfully applied to image the stress field in a PMMA sample. An attractive and original feature of the method is that the optical field at the surface of the sample can be simultaneously determined in amplitude, phase, and polarization state with a single image acquisition. This is, to our knowledge, a unique feature of the method presented here.

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References