

Wave-front reconstruction from multidirectional phase derivatives generated by multilateral shearing interferometers

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Received July 19, 2004

To increase the accuracy of wave-front evaluation, we propose to exploit the natural capability of multiple lateral shearing interferometers to measure simultaneously more than two orthogonal phase derivatives. We also describe a method, based on Fourier-transform analysis, that uses this multiple information to reconstruct the wave-front under study. © 2005 Optical Society of America

OCIS codes: 120.2650, 120.3180, 120.3940, 120.5050, 140.3300.

Lateral shearing interferometry is commonly used by the optics community to test lenses and laser beams and to control adaptive optics. Indeed, it offers the crucial advantage that it yields an analyzed wave front without the use of a reference wave. Usual lateral shearing interferometers (LSIs) generate only one replica of an analyzed wave front,¹ and analysis of the obtained interference fringes can only lead to the wave-front derivative in the direction of shear. To avoid error propagation, reconstruction methods based on the least-squares estimation require the wave-front derivative in two orthogonal directions. Thus, with these LSIs, two measurements must be made separately along two orthogonal directions of shear.²

Recently, a new family of LSIs that we call multi-LSIs was developed. They are based on interference of more than one replica of an analyzed wave front with different directions of shear. In this family there are several types: (a) the cross-grating LSI,³ which is used in diffraction-limited extreme-ultraviolet optics; (b) the three-wave LSI,⁴ which is largely used for intense laser beam evaluation, correction, and shaping^{5–9}; (c) the modified Hartmann mask (MHM),^{10,11} which is devoted to laser beam evaluation; and (d) the Shack–Hartmann wave-front sensor,¹² which uses an array of microlenses, which is prevalent in the domains of adaptive optics or ophthalmic evaluation. It is not common to consider this last setup a LSI. However, considering that the array of microlenses is a bidirectional phase grating,^{13,14} the Shack–Hartmann wave-front sensor enters naturally into the family of LSIs described here.

To show the capability of multi-LSIs to measure more than two orthogonal derivatives and for simplicity, let us consider the MHM, which diffracts four replicas of the incoming wave front in a Cartesian geometry. A schematic interference pattern of this configuration is shown in Fig. 1. The advantage of such a geometry is that the wave-front derivatives in the two usual orthogonal directions x_1 and x_2 can

be measured simultaneously because of the separate study of the interference of the two couples of beams sheared along x_1 and then along x_2 . Nevertheless, considering the global interference of the four beams, information on the wave front according to the cross directions (x_3 and x_4) appear naturally. Finally, one can exploit four derivatives in the four directions of shear to obtain the wave front. This thought process can be generalized to other multi-LSIs, and so the study of their interferogram can provide wave-front derivatives in multiple directions of shear. We propose to use these additional derivatives to improve the accuracy of reconstruction.

To detail our method of measurement and for simplicity, we first consider a multi-LSI without taking

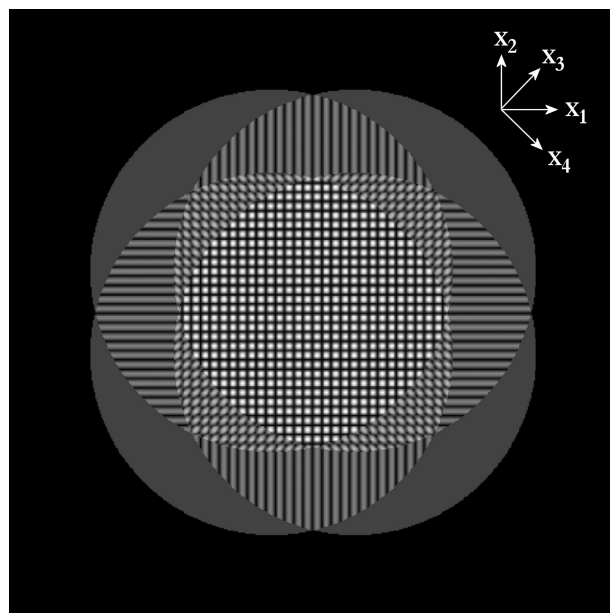


Fig. 1. Schematic interference pattern of four replicas in a Cartesian geometry and definitions of the shear directions.

into account the boundaries of the intensity profile. The analysis of the interference pattern leads to n derivatives of an analyzed wave front W in n different directions x_n .¹⁵ The derivative in the j th direction will be noted G_{x_j} . For each direction x_j , the Fourier transform of the derivative is given by

$$\tilde{G}_{x_j} = 2i\pi u_j \tilde{W}, \quad (1)$$

where $\tilde{W}$ is the Fourier transform of W and u_j is the conjugated variable of x_j in the spatial frequency domain. To obtain an estimate $\tilde{W}_e$ of $\tilde{W}$, we calculate a quadratic cost function by use of

$$E(\tilde{W}) = \sum_j |\tilde{G}_{x_j} - 2i\pi u_j \tilde{W}|^2. \quad (2)$$

The quantity $\tilde{W}_e$ is computed as the minimizer of this cost function and is given by

$$\tilde{W}_e = \frac{-i}{2\pi} \frac{\sum_j u_j \tilde{G}_{x_j}}{\sum_j u_j^2}. \quad (3)$$

In theory, for finite support, extrapolation of the wave front beyond the boundaries by use of a Gershberg-type algorithm¹⁶ must be done, as already applied by Roddier and Roddier¹⁷ in the particular case in which only two orthogonal derivatives are available. In practice, if the number of measurement points is large, then the noise caused by the support often becomes negligible compared with the noise of the derivatives. In this case, the step of extrapolation can be avoided.

Another method of reconstruction proposed by Legarda-Sáenz *et al.*¹⁸ can be used. Their algorithm is based on the least-squares method and estimates the wave front from several directional derivatives of itself obtained by multiple acquisitions of fringe patterns with different displacement vectors.

We applied the reconstruction technique expressed by Eq. (3) to a numerical example in which an aberrated wave front [see Fig. 2(a)] impinges on a MHM. In this case, four derivatives are available and can be numerically extracted from the Fourier transform of the deformed interferogram in the overlap region of the four beams.¹⁵ In practice, they are computed by means of a judicious selection of four harmonics in that spectrum (see Fig. 3). In this particular case the harmonics along u_1 and u_2 are twice as high as those along u_3 and u_4 , and the derivatives deduced from the study of harmonics along u_3 and u_4 are multiplied by $\sqrt{2}$ in comparison with those measured in the x_1 and x_2 directions because of the larger shear distance in the x_3 and x_4 directions. The signal-to-noise ratio of the derivatives along x_3 and x_4 is then $\sqrt{2}$ times smaller than the signal-to-noise ratio of the derivatives measured along the x_1 and x_2 directions. To take into account this decrease, we can weight the measured derivatives in terms of its signal-to-noise ratio in our reconstruction. The algorithm presented above is applied to these weighted derivative maps. Assuming white centered noise in

the computed interferogram, two reconstructions have been made [shown in Figs. 2(b) and 2(c)]: first from the two usual orthogonal derivatives as is classically done (according to x_1 and x_2) and then from the four available derivatives. Figure 2(d) shows histograms of the error on these two reconstructed wave fronts. The curves show that the noise decreases because of the use of more than two derivatives, as discussed by Legarda-Sáenz *et al.*¹⁸ In the example presented here, the fact that four derivatives are used instead of two leads to a noise reduction of $\sim 18\%$.

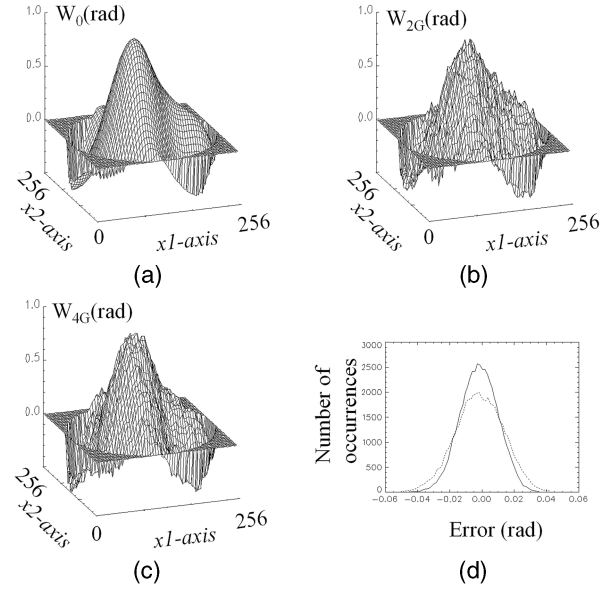


Fig. 2. (a) Computed impinging wave front W_0 , (b) reconstructed wave front with two orthogonal derivatives W_{2G} , (c) reconstructed wave front with four derivatives W_{4G} , (d) histograms of the difference between the noiseless and the reconstructed wave fronts with two orthogonal derivatives (dashed curve) and with four derivatives (solid curve).

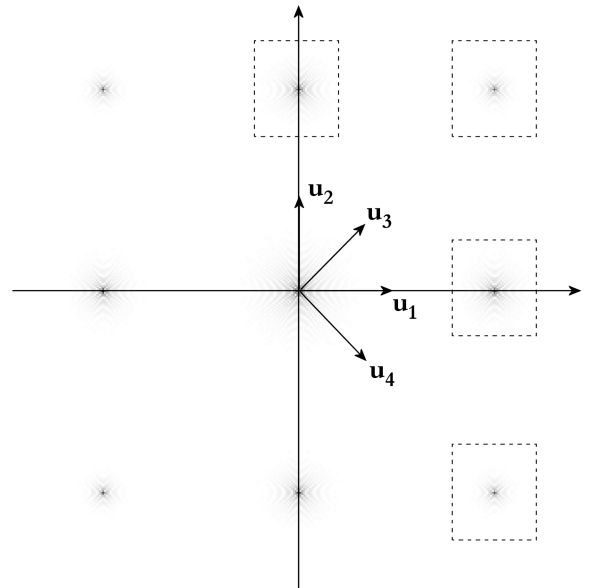


Fig. 3. Fourier transform of the interferogram produced with a MHM.

In conclusion, we have reported that a new family of LSIs has the natural capability to measure simultaneously more than the two orthogonal derivatives that are commonly used to reconstruct the wave front. This capability is characterized by the presence of many harmonics in the interferogram spectrum. We have therefore proposed a method of reconstruction that takes into account all the *de facto* information included in the interferogram, which reduces the noise of the reconstructed wave front. Note that, for each multi-LSI a specific strategy, taking into account the signal-to-noise ratio of each harmonic, must be applied to obtain the optimal reconstruction.

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